[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1427 **C**

Unique Paper Code : 32351303

Name of the Paper : BMATH 307 – Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory
3. Attempt any **Five** questions from each section. **All** questions carry equal marks

SECTION I

1. Let $f(x,y) = \dfrac{xy\left(x^2 - y^2\right)x}{x^2 + y^2}$ if (x, y) ≠ (0,0)

$= 0$ otherwise

Show that f(0, y) = –y and f(x, 0) = x for all x and y.

P.T.O.

2. Use incremental approximation to estimate the function $f(x, y) = \sin(xy)$ at the point

$$\left(\sqrt{\frac{\pi}{2}}+.01, \sqrt{\frac{\pi}{2}}-.01\right)$$

3. If $z = xy + f(x^2 + y^2)$, show that $y\,\partial z/\partial x - x\,\partial z/\partial y = y^2 - x^2$.

4. Assume that maximum directional derivative of f at $P_0(1,2)$ is equal to 50 and is attained in the direction towards $Q(3, -4)$. Find ∇f at $P_0(1, 2)$.

5. Find the absolute extrema of $f(x, y) = 2x^2 - y^2$ on the disk $x^2 + y^2 \leq 1$.

6. Use Lagrange multiplier to find the distance from (0, 0, 0) to plane $Ax + By + Cz = D$ where at least one of A, B, C is nonzero.

SECTION II

1. Compute the integral $\int_0^1 \int_x^{2x} e^{y-x}\,dy\,dx$ with the order of integration reversed.

2. Use Polar double integral to show that a sphere of radius α has volume $\frac{4}{3}\pi a^3$.

3. Compute the area of region D bounded above by line $y = x$, and below by circle $x^2 + y^2 - 2y = 0$.

4. Find the volume of the solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

5. Evaluate $\iiint_D \frac{dx\,dy\,dz}{\sqrt{x^2+y^2+z^2}}$, where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

6. Use a suitable change of variables to find the area of region R bounded by the hyperbolas $xy=1$ and $xy=4$ and the lines $y=x$ and $y=4x$.

SECTION III

1. Find the mass of a wire in the shape of curve C: $x = 3 \sin t$, $y = 3 \cos t$, $z = 2t$ for $0 \leq t \leq \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = x$.

2. Find the work done by force

$$\vec{F}(x,y,z) = \left(y^2 - z^2\right)\hat{i} + (2yz)\hat{j} - \left(x^2\right)\hat{k}$$

on an object moving along the curve C given by $x(t) = t,\; y(t) = t^2,\; z(t) = t^3,\; 0 \leq t \leq 1.$

3. Use Green's theorem to find the work done by the force field

P.T.O.

$$\vec{F}(x,y) = (3y - 4x)\hat{i} + (4x - y)\hat{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

4. Use Stoke's theorem to evaluate the surface integral

$$\iint_S \left(\text{curl } \vec{F}.N\right) dS$$

where $F = x\, i + y^2 j + z\, e^{xy}\, k$ and S is that part of surface $z = 1 - x^2 - 2y^2$ with $z \geq 0$.

5. Use divergence theorem to evaluate the integral

$\iint_S \vec{F}.N\, dS$ where $\vec{F}(x,y,z) = (\cos yz)\hat{i} + e^{xz}\hat{j} + 3z^2\hat{k}$,

where S is hemisphere surface $z = \sqrt{4 - x^2 - y^2}$ together with the disk $x^2 + y^2 \leq 4$, in x-yplane.

6. Evaluate the line integral $\int_C \vec{F}.d\vec{R}$

Where $\vec{F}(x, y) = [(2x - x^2y)\, e^{-xy} + \tan^{-1} y]\,\hat{i} + \left[\frac{x}{y^2+1} - x^3 e^{-xy}\right]\hat{j}$ and C is the ellipse $9x^2 + 4y^2 = 36$.